Chapter 13

Synchrony and Precise Timing in Complex Neural Networks

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13.1. Precise Timing in the Brain?

Dynamical processes in the human brain underlie all our perceptions, actions and intellectual abilities, such as the formation of memories and the construction of high-level knowledge about the world. It is generally assumed that the large number of neurons that collectively interact in networks – and not the high diversity of the individual neurons – support the various functions of the brain. Understanding these collective processes thus constitutes a fundamental current problem of research, with key consequences also in disciplines outside neuroscience, ranging from psychology to the social sciences and philosophy. Moreover, most theoretical investigations of the collective dynamics of neural networks require new and advanced methods originally used in similar form in theoretical physics, computer science and mathematics. Often these methods are to be newly developed and thus also initiate complementary lines of research in these disciplines.

Neurons are connected to networks via synapses and communicate with each other using short-lasting ($\approx$ 1ms) electrical pulses called action potentials or spikes [79, 126]. These spikes are predominantly sent via chemical synapses to other neurons which process the incoming signals, change their internal state in response, and in turn send spikes at state-dependent times. The chemical processes involved are comparatively slow (several milliseconds) and cause a significant delay in signal transmission [79]. Chemical synapses are also highly adaptive and their strengths can be changed during learning processes.

Several basic functional features of neuronal networks, such as the selectivity of visual cortical neurons to oriented bar stimuli [126], are well characterized by spa-

tially and temporally coarse-scale quantities, e.g. by the number of spikes emitted by a local population of neurons in a larger time interval (their spike rate) [33]. Nevertheless, there is accumulating evidence [6, 44, 45, 55, 121, 130] that the timing of spikes may be highly coordinated between neurons and play a role in neural processing as well. Neurons that spike coincidentally within a few milliseconds, or with a precise time lag between them have been observed in different neuronal systems [6, 44, 67, 87, 109, 119, 121, 130]. Coincident spiking can occur with high statistical significance correlated to internal states of the brain [121, 130] and patterns of spikes may re-occur repeatedly and in a particular order (second order spike patterns) [44, 45]. Patterns of precisely timed spikes and synchronization in the millisecond range are therefore discussed to be essential for information processing in the brain [4–6, 17, 31, 96, 117, 124, 130]. For a number of physiological experiments [6, 67, 87, 130] however, the statistical significance of some of the findings is currently highly debated [14, 99, 109, 119]. It has been argued that in some experiments [6, 87, 130] the significance of the occurrences of spike patterns highly depends on the underlying statistical assumptions about the spike trains [14, 119]. Further, [109] shows that the occurrence of repeated dynamical motifs of the membrane potential (which were assumed to indicate and generate spike patterns) is equally likely in random or randomized sub-threshold dynamics if the randomly generated membrane potential has similar coarse statistical properties (such as the power spectrum) as the actually measured one. It is thus still an open problem whether and how neurons may precisely coordinate their spiking activity across complex networks, and which role the identity of individual neurons and their inter-connectivity actually play.

Below we present two classes of hypotheses that may explain the dynamical origin of patterns of precisely timed spikes and microscopic, inter-neuronal synchronization, i.e. non-random, coincident spiking. One hypothesis states that feed-forward anatomical structures are embedded in cortical circuits and support the propagation of synchronous spiking activity of groups of neurons that constitute the layers of the feed-forward architecture [4, 5, 38, 62]. This kind of dynamics was termed 'synfire chain' activity [4]. As in current physiological experiments only small subsets of neurons are observed, the synfire chain hypothesis permits the occurrence of spiking activity that is synchronized with millisecond precision as well as the persistence of spike patterns over longer time periods. A second, alternative hypothesis states that recurrent networks may collectively organize patterns of precisely timed spikes without the need of specific feed-forward anatomy. We will give more emphasis to this latter hypothesis as it is more recent and its theoretical aspects are only marginally described so far in standard references.

This chapter is organized as follows: In Section 13.2, we briefly present the key ideas underlying synfire chain dynamics and state the main results on this topic. The remainder of this chapter is devoted to recurrent network models. In Section 13.3 we introduce a class of analytically tractable models of spiking neural

networks that serves as our guide throughout; we also list related model classes as well as biophysically more detailed models. Section 13.4 gives an overview of basic states and important collective phenomena in recurrent spiking neural networks. In Section 13.5 we present recent approaches to characterize the emergence of patterns of coordinated, precisely timed spikes in neural network models. Finally, in Section 13.6, we conclude and highlight some open questions. To keep this overview concise, we focus on conceptual questions and theoretical challenges throughout, sometimes passing over technical subtleties and smaller (yet not unimportant) problems.

13.2. Feed-Forward Mechanisms: Synfire Chains

The synfire chain hypothesis states that precisely timed spiking in cortical networks is due to the existence of anatomical feed-forward structures that are part of cortical circuits [4, 5]. Thinking abstractly, such a feed-forward structure can be separated into groups of neurons or 'layers', such that neurons in one layer receive many synaptic connections from neurons in the previous layer (Fig. 13.1a). In its simplest setting, the connectivity between layers is uni-directional and global, i.e. each neuron in a layer receives an excitatory synapse from every neuron in the previous layer. In general, this connectivity between layers is diluted and only predominantly excitatory; still, when many spikes are received from a sufficiently synchronized pre-synaptic group of neurons, the likelihood that a post-synaptic neuron generates a spike is increased.

If now some initial layer of neurons emits spikes synchronously, i.e. with only small inter-neuronal variations (on the order of one millisecond) each of the post-synaptic neurons in the next layer receives an almost synchronous collection of spikes ('volley') after an effective transmission delay [79]. Collectively, this may initiate synchronous spiking activity generated by that next layer. Depending on the temporal spread of the spikes, on the inter-connectivity between layers and on the total number of synchronously firing neurons in a group, this may lead to the persistent (or decaying) propagation of synchronous activity along the chain [38, 62], (Fig. 13.1b). The feed-forward anatomy underlying synfire chains is viewed as an embedded part of a larger recurrent circuit such that each neuron receives in addition many synaptic inputs from outside the chain; as the basic state of the entire circuit is often asynchronous and irregular [33, 138, 140, 162], this additional input is typically regarded as noise that adds to the propagating synchronous activity [4, 5, 38, 53, 61, 62, 83, 101, 159].

Already in 1963 Griffith [56] suggested that densely coupled feed-forward anatomy may have a functional role in the brain. He investigated the capability of what he calls 'transmission lines' to reliably transmit information in a non-trivial way along feed-forward chains of abstract units. This idea was refined by Abeles [4, 5] to account for the appearance of precise spiking sequences of neurons. Diesmann and others showed that both fully connected and randomly diluted

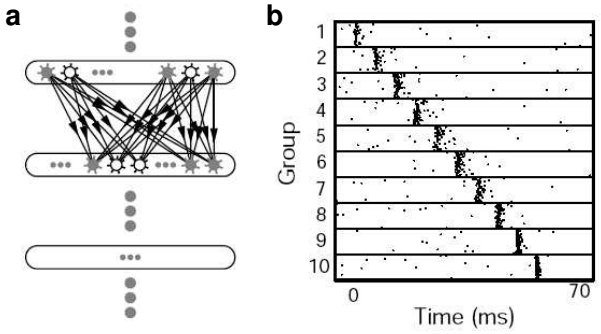

Fig. 13.1. Synfire chain anatomy (a) and dynamics (b). (a) Chain of groups of neurons connected in a feed-forward manner. (b) Activity synchronizes further as it propagates along the chain. (Modified from [38].)

feed-forward structures permit the stable propagation of synchrony [7, 38, 53, 62, 145, 159]. Model studies of synfire chains that are actually embedded in recurrent networks (rather than treating the influence of the embedding network just as additional noise) show that persistent propagation of localized synchronous events within recurrent cortical circuits is not simple to realize [83, 101, 146]. In particular, extensive numerical studies show that although propagation of synchrony can be achieved, often pathological dynamics occur, for instance synchronous activity which spreads and covers the entire network after a short time ('synfire explosion'), or synchronous activity in the embedded chain dies out quickly due to inhibition from the embedding network [12, 101, 146]. It has been shown recently that depending on the features of individual neurons and on the network architecture, stable propagation of synchrony along chains embedded in recurrent networks may also be achieved in a robust way [83]. Another possibility to construct networks with feed-forward pathways is to strengthen those connections that are already present in a recurrent random network [164]. However, strong amplifications of synapses and specific changes in the response properties of neurons along the pathway are required to enable the propagation of synchrony over a few groups. A mechanism that might enable persistent synchronous activity in embedded architectures with moderately strong pathway structure is nonlinear enhancement of synchronous inputs due to dendritic spikes that was recently found in neurophysiological experiments ([9, 46, 47, 116, 122], cf. also [106]).

Successive excitation of neurons in groups with distributed transmission delays can generate spiking activity that is not synchronous, but precisely time-lagged (with the lag defined by the transmission delays), resembling synfire chain dynamics. Works by Izhikevich and coworkers [68, 69] show that such groups of neurons with strong coupling can spontaneously form in a random network due to spike timing dependent plasticity (see, e.g., [33, 79]) and that they generate detectable spike patterns with millisecond precision although embedded in a larger network. Taken

together, current theoretical knowledge supports the synfire hypothesis in systems with specific constraints on synaptic dynamics, single neuron features, neural interactions, and inter-connectivity between groups.

The current stage of experimental research is inconclusive. Despite some interesting studies which might support the synfire chain hypothesis [94, 113, 125, 139], there is no key experiment that directly proves – or disproves – the existence of synfire chain anatomy or dynamics. Such an experiment would require either a large-scale structural investigation of local cortical anatomy, proving or excluding the necessary non-random feed-forward connectivity; or a large-scale dynamical study, recording spikes of a large number of neurons simultaneously and repeatedly under controlled conditions, such as to explicitly show (or exclude) the existence of synchronous activity propagating along fixed paths.

13.3. Recurrent Neural Networks

Alternatively, in recurrent networks without specifically embedded feed-forward structures, mechanisms other than synchronous excitation along feed-forward anatomy might generate spikes that are precisely coordinated in time and among different neurons. For recurrent networks, however, theoretical investigations that take into account individual neurons' spike times and thus go beyond mean-field descriptions are often highly non-standard. Thus up to date precise timing of spikes in recurrent networks is far less understood than synfire chain dynamics. Conceptual challenges include the nonlinear features of individual neurons and their interactions, the complex recurrent connectivity of the networks, the existence of transmission delays that make the dynamical systems formally infinite-dimensional, and strong heterogeneities that might be present among the neurons and their interactions.

To cope with these challenges, many studies have focused on networks of idealized model neurons, e.g. of integrate-and-fire type [33, 51, 71, 86, 120]. In the following, we introduce a class of spiking neural network models for which a wide range of dynamical phenomena becomes analytically accessible. We briefly list related model classes and biophysically more detailed models at the end of this section and describe some basic and more involved dynamical states of spiking activity in the subsequent sections.

13.3.1. *An analytically accessible class of models*

Consider a network of $N \in \mathbb{N}$ neurons that interact by sending and receiving spikes (see, e.g., [40, 60, 108, 149]). The state of each neuron j at time s is specified by a single real variable, the membrane potential $V(s)$ that evolves according to

$$\frac{d}{ds}V_j(s) = g(V_j(s)) + \sum_{i=1}^{N}\sum_{m\in\mathbb{Z}} \varepsilon_{ji} K(s - (s_i^m + \tau_{\mathrm{V}})) \tag{13.1}$$

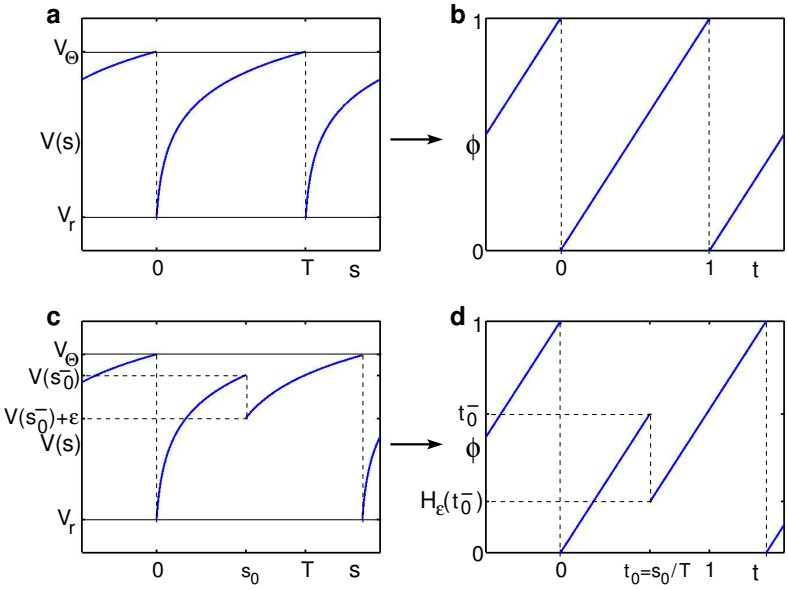

Fig. 13.2. Dynamics of membrane potentials and phases. Upper panels show free evolution of potential (a) and corresponding phase (b). Lower panels show evolution of potential (c) and phase (d) with an inhibitory spike arriving at time s_0. (Modified from [81].)

where $g(.) > 0$ specifies the local dynamics of neuron j, ε_{ji} denotes the strength of synaptic coupling from neuron i to neuron j, and s_i^m specifies the time neuron i sends its mth spike. When a neuron i reaches a potential threshold $V_i((s_i^m)^-) = V_\Theta$, its potential is reset to $V_i(s_i^m) = V_r$ and it sends a spike which is received by the postsynaptic neurons j after a delay time $\tau_V > 0$. Here, $K(.)$ is a response kernel that determines the post-synaptic current in response to an incoming spike signal. Such a kernel satisfies $\int_{-\infty}^{\infty} K(s)ds = 1$ and $K(s) = 0$ for $s < 0$. Often one considers the limiting case of fast synaptic response, $K(s) = \delta(s)$. For such systems the smooth dynamics of the neurons is interrupted by two kinds of events that occur at discrete times only: sending of spikes (and reset) and receiving of spikes. This results in a hybrid dynamical system [13, 19, 133] with continuous-time dynamics interrupted at discrete times where maps are applied [10, 23].

A universal representation of the network dynamics provides elegant analytical access to state space trajectories. The network of spiking neurons (13.1) with $K(s) = \delta(s)$ is equivalently described by the dynamics of phase variables $\phi_i(t) \leq 1$ with rescaled time variable $t = s/T$: The free solution $\tilde{V}(s)$ of Eq (13.1) in the absence of coupling (all $\varepsilon_{ji} = 0$) through the initial condition $\tilde{V}(0) = V_r$ increases monotonically and is assumed to reach the threshold after a time T such that $\tilde{V}(T^-) = V_\Theta$. This free solution defines a bijective map (Fig. 13.2a,b)

$$U : (\phi_-, 1] \to (V_-, V_\Theta]; \phi \mapsto U(\phi) := \tilde{V}(\phi T)\,, \tag{13.2}$$

between potential and phase representation via a continuously differentiable 'rise function' $U(\phi)$ that is monotonic increasing because $g(.) > 0$ in Eq. (13.1). In Eq. (13.2), V_- and ϕ_- are possible lower bounds of potential and phase; if there are no bounds in potential or phase, $V_- = -\infty$ or $\phi_- = -\infty$, cf. [74, 102, 103]. In the absence of interactions, the phases increase uniformly and obey $d\phi_i/dt = 1$. When ϕ_i reaches its phase threshold $\phi_i(t^-) = 1$ it is reset to $\phi_i(t) := 0$ and a spike is sent such that $t = t_i^m = s_i^m/T$. This spike is now received by the post-synaptic neurons j after a rescaled delay time $\tau = \tau_{\rm V}/T$, where it causes an instantaneous phase jump (Fig. 13.2c,d) according to

$$\phi_j(t+\tau) = H_{\varepsilon_{ji}}(\phi_j((t+\tau)^-)) \tag{13.3}$$

mediated by the transfer function

$$H_\varepsilon(\phi) = U^{-1}(U(\phi)+\varepsilon) \tag{13.4}$$

that is strictly monotonic increasing both as a function of ε and ϕ, because $U' > 0$ [74, 102, 103]. If $U(\phi)+\varepsilon$ may exceed the codomain of $\tilde{V}$ and thus the domain of U^{-1}, this has to be accounted for by case distinctions in the definition of H_ε [40, 102, 103, 151]; for the sake of conciseness, we will not discuss this complication here. If the coupling is excitatory ($\varepsilon > 0$), it is phase-advancing, $H_\varepsilon(\phi) > \phi$, enabling the neuron to emit its next spike earlier than without that coupling; if it is inhibitory ($\varepsilon < 0$), it is phase-retarding, $H_\varepsilon(\phi) < \phi$, such that the neuron will emit its next spike later than without that coupling.

As demonstrated before [40, 74, 103, 104, 108, 150, 151] this phase representation allows for exact numerical integration and provides elegant analytic access to trajectories of the network dynamics, even if the local neuron dynamics is not characterized by a simple, e.g. linear, differential equation [108].

We remark that the above models are current-based, i.e. the coupling strengths ε_{ji} do not explicitly depend on the state of the post-synaptic neuron. Under certain conditions, networks with conductance based synapses (where the coupling strength depends on the state as $\alpha(V_j - V_{\rm rev})\varepsilon_{ji}$), or others where the interaction is explicitly state-dependent, can also be modeled using a phase description together with a modified rise function [132, 151, 160]. Often models of spiking neural networks are formulated in a generalized way, with various kinds of heterogeneities in the neuron dynamics, in the delay times and with temporally extended interaction kernels $K(.)$ that may depend on the synaptic connection and the state of the post-synaptic neuron at reception time. Moreover, additional noise and driving forces may add to the recurrent network dynamics (13.1). For simplicity of presentation, we will not describe these features here in detail, but only refer to the respective literature where appropriate.

The model class includes, among others, the simple non-leaky integrate-and-fire (IF) model where $g(V) = $ const. and thus $U(\phi) = \phi(V_\Theta - V_r) + V_r$ [50, 100, 110], standard leaky IF models ($g(V) = I - \gamma V$) [1, 27, 86, 155], quadratic integrate-

and-fire (or theta-) neurons ($g(V) = V^2 + I$) [39, 43, 71, 114], abstract neural oscillator models where $U(\phi) = b^{-1} \ln(1 + (\exp(b) - 1)\phi)$ [36, 108], and exponential IF neurons where $g(V) = I - \gamma V + \exp(\beta V)$ [42, 43, 129]. The model class is limited by the idealization that the sub-threshold neuron dynamics is well characterized by a single variable; moreover, whereas an analytic approach is conceptually simple in the limit of infinitely fast response (where the kernel is a delta-distribution), it typically becomes restricted for certain post-synaptic response kernels that are temporally extended.

13.3.2. *Related models*

Besides the standard model class defined via (13.1), several variants are widely used as well. Often, additional degrees of freedom are introduced. One phenomenological class of models has a spike-triggered adaptation variable [22, 70, 71, 77]. In dependence of the parameters, neurons of this class show a wide spectrum of qualitative features observed in biological neurons and at the same time allow fast numerical simulations of large networks if the individual neurons' features are well understood [70, 71]. It has the disadvantages that its dynamics is analytically accessible only in rare special cases, and in numerical simulations, though they are much faster than, say for Hodgkin Huxley neurons (see below), the dynamical parameters are similarly hard to restrict. The 'spike response model' works in the original potential representation and includes additional refractoriness or adaptation, modeled as a threshold dynamics that is not present in (13.1). Recent works [22, 76, 77] suggest that certain representatives of spike response model neurons with adaptation well reproduce the response of real neurons to specific random current inputs.

Spiking neural network models with temporally extended interactions often also characterize the response dynamics by one additional degree of freedom per neuron, e.g. by a second differential equation, which is, however, usually chosen to be solvable in closed form such that Eq. (13.1) is regained (see, e.g., [3, 154, 168]). Biophysically more detailed models, such as the Hodgkin-Huxley, Morris-Lecar, Fitzhugh-Nagumo, or Hindmarsh-Rose models ([64, 65, 111], see [71] for a comprehensive review) require several dynamical variables and many physiological parameters for each neuron. As such they are appropriate for modeling dynamical network aspects of well-known systems (see, e.g., [35]); at the same time, they typically preclude analytical arguments and for many systems it is unclear how to suitably restrict all model parameters or even whether the chosen model is appropriate at all [115].

Whereas all deterministic models have their variants that include additional stochastic influences, modeling, e.g., synaptic failure [80] or local noise induced by ion channels [15], intrinsically stochastic models may sometimes be more appropriate for the description of single neuron or network dynamics [49, 78, 88, 127, 128, 143, 144, 156]. For instance, Levina *et al.* [88, 89] have recently shown that the dynamics of branching processes under certain conditions well describe the stochas-

tic dynamics of large recurrent networks. Of course in all the above-mentioned frameworks, also additional features may be studied, including synaptic plasticity on short [2, 88, 93, 153] and long time-scales [63, 68, 90, 112], ion channel cooperativity [115], compartmental or spatially extended structure [20, 82, 135] and non-additive features of the interactions [9, 88, 93, 106, 115, 153].

Moreover, various models of abstract rate-coded often binary-state or discrete-time neurons [16, 66, 91, 92, 131, 165] exist that are valuable for studying conceptual problems of computation or information processing in neural systems, but by their very nature generically do not capture the precise timing of spikes. Very recently the link between continuous-time and discrete-time models has been reconsidered with the interesting resulting suggestion [29, 30] that under certain conditions specific discrete time models may actually more appropriately describe the spiking dynamics of recurrent networks.

13.4. Basic Collective States of Recurrent Networks

Here we provide a brief overview on basic collective states of deterministic recurrent networks. This should pave the way to a better understanding of the concepts and the complex spatio-temporal dynamics presented in the next section.

13.4.1. *Quiescence and synchrony*

Obviously, the simplest dynamical state of a spiking neural network is global quiescence, where no neuron is emitting any spike. This is a trivial network state because its dynamics is just the collection of all individual neuron dynamics, even in the presence of driving signals and fluctuations. Still it is sometimes valuable to know under which conditions the quiescent state exists and is stable, for instance if the emergence of a complex, persistent state from the quiescent one (or from an almost quiescent one) is to be understood [57, 59].

In fact, single neurons are intrinsically excitable systems and typically quiescent if not driven by synaptic inputs, external currents, fluctuations or by other means. Therefore, the modeling of single spiking neurons ranges essentially between two extreme limits (see, e.g., [155, 156]). One limit is stochastic: neurons receive independent stochastic sequences of spikes (e.g. Poisson spike trains) and therefore also generate stochastic spiking dynamics theirselves (e.g. [27, 28, 156]). The second limit is deterministic: neurons receive sufficiently strong temporally uniform input currents such that their dynamics becomes tonic periodic spiking. We here focus on deterministic models of spiking neural networks. As we will see below, however, these neurons collectively still often exhibit dynamics that resembles random processes [58, 74, 106, 162, 163].

Arguably the simplest non-trivial and truly collective state is the fully synchronous state, a periodic orbit in which every neuron emits spikes periodically and at the same times as all the other neurons in the network. It is dominant in globally

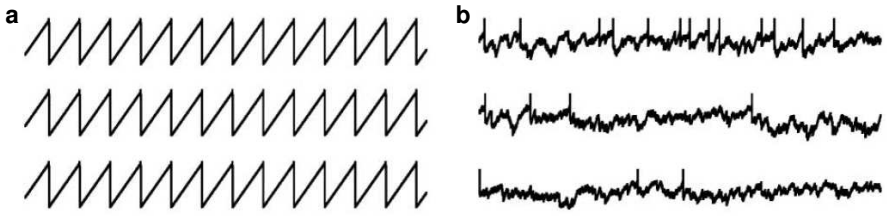

Fig. 13.3. Synchronous (a) and irregular (b) dynamics of three neurons in a sparse random network [149]. Both dynamics may coexist in the same network and external stimulations can induce switching between them, cf. also Fig. 13.4. (Modified from [149].)

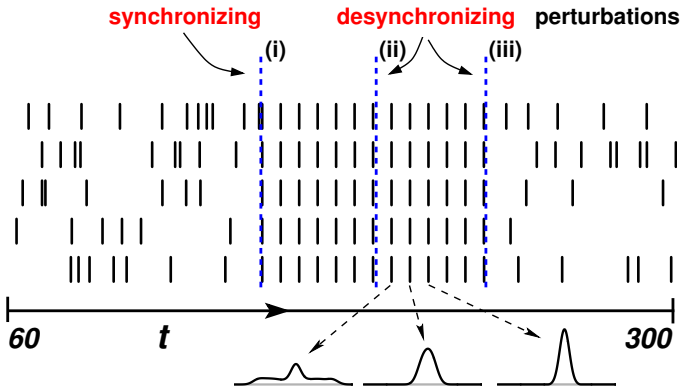

Fig. 13.4. Transitions between irregular and synchronous states due to external input signals. Synchronized excitatory external input (i) causes the dynamics to assume the synchronous state. The synchronous state is stable: A sufficiently small desynchronizing perturbation (ii) does not lead to the irregular state. Only after strong desynchronizing signals (iii), e.g. induced by a large number of random inhibitory and excitatory input spikes, the system switches back to the irregular state. (Modified from [149].)

excitatorily coupled networks of leaky (and non-leaky [136]) IF-like neurons if the interactions exhibit zero delay ([108], see also [52]), but it is unstable in the presence of arbitrarily small delays in globally and more complex connected excitatory networks [40, 41, 148, 149]. If the coupling is inhibitory, however, synchrony might occur and even be predominant [106, 149, 152, 161] in the presence of interaction delays as well, cf. Figs. 13.3a, 13.4. Noise in such systems affects synchrony in a non-trivial way [26].

In globally coupled or spatially extended homogeneous networks of spiking units also less symmetric solutions exist, including waves [21], periodic localized activity [134] and cluster states [40] exist that are well known from smoothly coupled systems.

13.4.2. *Away from synchrony: first hints towards spike patterns in recurrent networks*

Systems that exhibit full synchrony, with all units obeying identical dynamics, necessarily are constraint, e.g. they exhibit some invariance. For instance, when considering networks of identical inhibitory neurons, the total input strength to each neuron in a complex network needs to be the same for the fully synchronous state to exist [148, 149]. Such an idealized condition is atypical for biological networks in which the total synaptic strengths may be roughly the same due to homeostasis [32, 157, 158], but at least weak inhomogeneities are prevalent. Naturally, weak inhomogeneities and the presence of other less idealized features will induce states similar to the fully synchronous one [3, 36, 154]. Stronger heterogeneities typically lead to states that are very distinct from synchrony and sometimes completely asynchronous [36].

Already in large networks of all-to-all and homogeneously coupled excitatory neurons with temporally extended synaptic responses, a partially synchronous state exists for a certain range of temporal extent [160]. In this partially synchronous state the total network firing rate oscillates periodically whereas the individual neurons send spikes quasi-periodically. This result by van Vreeswijk provides one possible mechanism for oscillations in neural circuits and at the same time is of interest mathematically as the local quasi-periodic activity adds up to global periodic activity. Brunel *et al.* [24, 25, 48] showed that also sufficiently strong inhibitory interactions can lead to high frequency network oscillations where the individual neurons fire irregularly and with low frequency. This type of dynamics has recently been proposed to underlie high frequency oscillations of Purkinje cells in the cerebellum [34].

Tsodyks, Mitkov and Sompolinsky uncovered a different interesting state similar to synchrony [154]; for globally and excitatorily coupled neurons with temporally extended synaptic responses arbitrarily weak inhomogeneities in the individual neurons' intrinsic time scales may split the neurons into two sub-populations (Fig. 13.5), one sub-population consisting of the slower neurons, that stay identically synchronized forever, and a second consisting of the intrinsically faster spiking neurons, which also have collective frequencies that are different from each other and larger than in the synchronized sub-population. We remark that already in such states, the timing of spikes of the synchronized sub-population is highly precise, despite inhomogeneities in the individual neuron features; moreover, the timing of spikes of neurons in the unlocked sub-population is relatively precise and close to that of the locked sub-population, for repeated, long stretches of time.

Networks with more complex connectivity may exhibit additional collective dynamical features induced by heterogeneities. Recent work [36] has shown that in networks of inhibitorily coupled IF-like neurons with delayed interactions, weak inhomogeneities in the coupling strengths (or equivalently, in other system parameters) induce a state close to full synchrony that exhibits well-defined patterns of

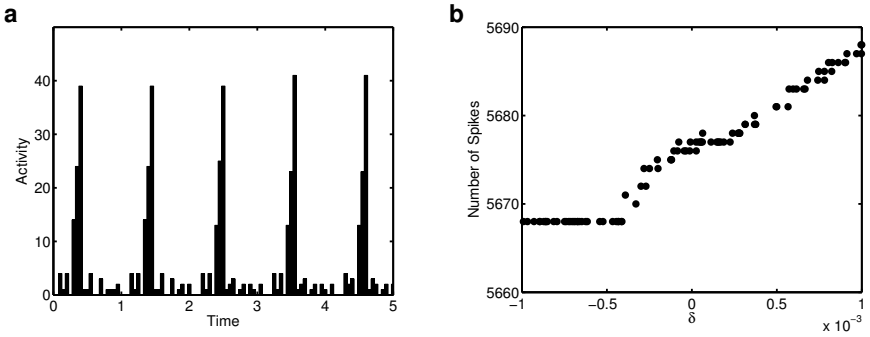

Fig. 13.5. Network activity and rates of individual neurons in a network of excitatory neurons with weak inhomogeneities as studied in [154]. The neurons split into two populations, neurons that are constantly phase-locked with zero lag and neurons which are not phase-locked. The network activity (a) shows large peaks of well synchronized firing together with some non-synchronous activity. The rate profile (b), displaying the rate of the neurons versus the perturbation of their driving, shows that there is a driving I_c separating phase locked neurons (driving $I < I_c$) and non-phase locked neurons (driving $I > I_c$).

spiking activity coordinated between the neurons. The exact analysis of spike times revealed also the transition point at which inhomogeneities become too strong such that states close to full synchrony (short patterns) cease to exist. Furthermore, the same work considered the occurrence of patterns of precisely timed spikes as an inverse problem (see sec. 13.5.2): For any given, predefined pattern of spikes that spreads over a sufficiently short time interval, Denker *et al.* [36] showed how to find the set of networks that exhibit that pattern as an invariant solution of its collective dynamics. Interestingly, in homogeneous sparsely connected random networks (or in those with sufficiently weak inhomogeneities) a synchronous (or almost synchronous) state may coexist [148, 149] with highly irregular asynchronous states (see Figs. 13.3 and 13.4 and also the next subsection).

Numerical investigations of inhomogeneous networks of inhibitory and excitatory sub-populations with delayed, temporally extended interactions [18] have shown that the two sub-populations may send spikes phase-locked but out-of-phase with each other, with all neurons in the separate sub-population close to synchronous with each other. As a sideline, that work suggests that patterns of locked spikes may occur also in neural circuits with a mixture of excitatory and inhibitory neurons; the mechanism underlying this phenomenon is similar to that described in Ref. [36] for purely inhibitory recurrent interactions.

13.4.3. *Asynchrony: Irregular, chaotic and balanced activity*

Besides simple synchronous states, asynchronous states provide a second type of basic activity in spiking neural networks. Depending on the features of the network considered, asynchronous states may predominantly emerge (i) as states in which neurons emit spikes individually and periodically and phase-locked to all other neu-

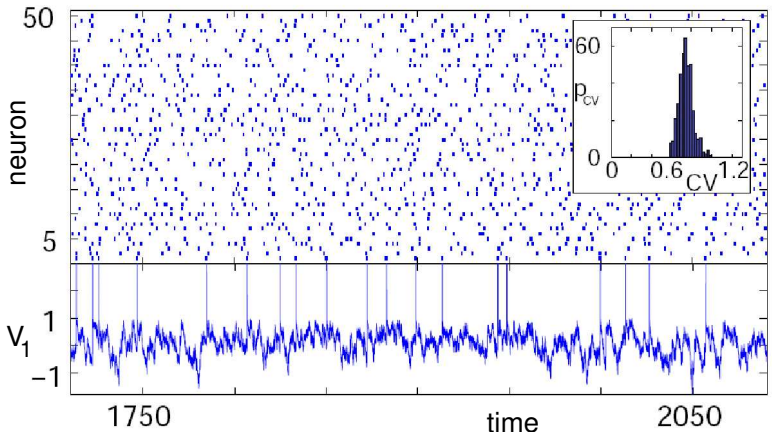

Fig. 13.6. Balanced irregular neural activity. Upper part shows irregular spiking dynamics characteristic for the balanced state. Lower part displays the normalized, highly irregular membrane potential of neuron 1. Inset displays the distribution of the coefficients of variation of the neurons' spike trains. This dynamics is stable against small perturbations, the irregularity does not result from chaos. (Modified from [74].)

rons in the network, see, e.g., [168], (ii) as non-periodic states with periodic oscillatory network rate dynamics (e.g. splay states [24, 25, 160] and cf. section 13.4.2) and (iii) and as aperiodic irregular states with constant network rate and no apparent coordination of spike times [24, 163].

The balanced state constitutes a key example of asynchronous irregular spiking as it is considered as a possible ground state of cortical activity. In such a state, excitatory and inhibitory synaptic input to each neuron balances such that the average membrane potential is sub-threshold and large fluctuations generate spikes at low rate and at seemingly random times [24, 162, 163]. Whereas the original assumption was that chaotic dynamics causes these apparently random spiking sequences, it was recently found ([73, 74], (cf. also [167]) that dynamics with the same irregularity is prevalent also in systems which do not exhibit chaotic, but rather stable microscopic dynamics. This raises the question which dynamical features actually generate asynchronous irregular spiking dynamics characteristic for the balanced state. A further question of current research is how balanced irregular activity may persist after a transient external stimulus has initiated it [57, 58, 84].

In theoretical investigations, the analysis of asynchronous states may also serve as a starting point to reveal mechanisms that underlie more coordinated neural activity by studying bifurcations away from asynchrony, see, e.g., [59].

13.5. Precise Timing in Recurrent Networks

As some results presented in the previous section already suggest, patterns of spikes that are precisely timed and coordinated among neurons may also emerge in the

collective dynamics of recurrent neural networks. Compared to patterns generated by feed-forward networks, this possibility so far is much less explored. Below, we present some recent developments where in part a detailed understanding of the collective phenomena is possible.

13.5.1. *Spike patterns as attractors of recurrent networks*

In certain networks that are dominated by inhibitory non-delayed interactions, the collective network dynamics converges to periodic spike patterns [75, 98]. These spike patterns are typically of low period and reached quickly such that they dominate the network dynamics on the relevant time scales. As shown before already for globally coupled networks [40, 41], interaction delays may have a drastic influence on the collective network dynamics. This is even more so if the network topology is complex and local dissipation becomes relevant, compare, e.g., [169] vs. [50, 108]. For instance, in inhibitorily coupled units delays strongly enhance the transient times towards periodic spike patterns [73, 74] such that stable irregular transients dominate the dynamics. Moreover, periodic orbits in these systems typically are long. In another example, very long delays induce switching between sequences [54] that recur several times and afterwards are non-recurrent. The results of Ref. [74] strongly suggest that the dynamics is nevertheless stable. An explanation for the switching phenomenon is detailed in [102].

13.5.2. *Realizing spike patterns in complex networks – An inverse problem*

Nearly all the above studies considered certain pre-specified networks of spiking neurons and studied what kinds of dynamics they may exhibit. In recent years, an inverse perspective was introduced [36, 97, 98, 103, 104, 123, 147, 166] where now the central question becomes "What kind of networks exhibit a given dynamics?". Such questions have been conceptually addressed two decades ago in abstract networks of non-spiking neurons [37, 66, 91, 92].

Prinz and coworkers [123] presented an extensive numerical analysis of three-neuron circuits and identified broadly distinct networks that exhibit nearly the same spiking dynamics. Makarov *et al.* [97] used stochastic optimization to find networks of given model neurons that most closely match observed spiking data. An analytical deterministic framework of network design was introduced recently [103, 104] to find the set of all possible networks that exhibit a predefined, e.g. periodic, spike pattern (Fig. 13.7). If a network solution exists at all (which it does under mild constraints), there typically is a high-dimensional set of networks that exhibit the same spike pattern as a possible dynamics. This set is parametrized, for instance, by the coupling strengths. The set is restricted by spike timing conditions, equations that impose the constraint that a given spike occurs where predefined, and by silence conditions, inequalities that ensure that a neuron does not emit a spike when none

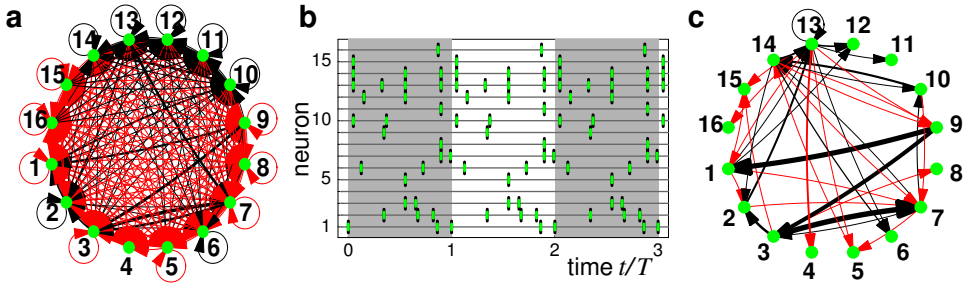

Fig. 13.7. Using the methods derived in [103, 104], networks with very different coupling statistics (a), (c) can be designed to generate the same pattern of spiking activity (b). Network (a) minimizes the L_2-norm ($\sqrt{\sum_{i,j} \varepsilon_{ij}^2}$) of the coupling matrix, network (c) minimizes the L_1-norm ($\sum_{i,j} |\varepsilon_{ij}|$). (Modified from [103].)

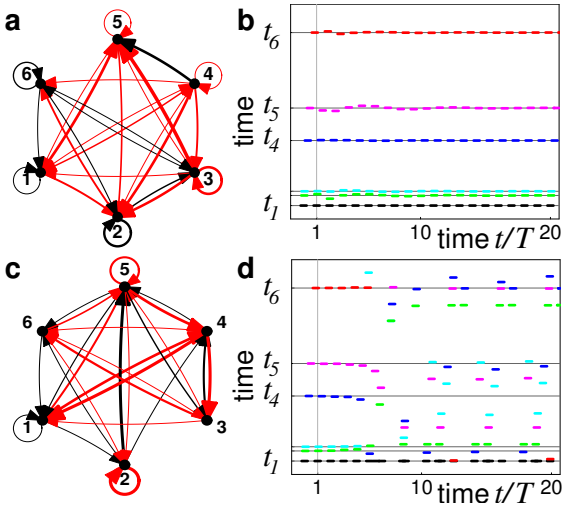

Fig. 13.8. The same periodic pattern of spikes can be realized as stable invariant dynamics (b) in one network (a) and as unstable invariant dynamics (d) in another network (c). (Reproduced from [103].)

is predefined. The equations and inequalities involve all parameters available in the class of model systems considered, including the network topology and the coupling strengths, the delay times and the local individual neuron dynamics [103, 104].

The high dimensionality of a typical solution space has important conceptual consequences. For instance, the same pattern may exist in networks with very distinct topologies and coupling types, cf. Fig. 13.7. Moreover, the same pattern may exist in networks with statistically similar topology but may be stable in one and unstable in another network, cf. Fig. 13.8. Besides numerical analyses, for large classes of networks even general analytical statements on the stability of such spike

patterns have been derived [102–105]. The stability properties are of particular interest, e.g. because they determine the computational capabilities of a network (where computation is not only possible with stable states [10, 11, 72, 95]) and because motifs that exhibit patterns which are stable (or unstable) might, if embedded in a larger network, influence the entire network's function in a specific way.

The theory of coupled phase oscillators suggests so-called chimera states as a possible link [118] between a fully synchronous state and asynchronous states. Chimera states were originally found [85] in rings of coupled identical limit-cycle oscillators with translation-invariant, non-local coupling. In chimera states, one sub-population of neighboring oscillators is phase-locked whereas oscillators in a second sub-population are asynchronous and neither locked with the first sub-population nor with each other. Such chimera states are thus similar to the partially synchronized states [154] described above. However, they are significantly different as they occur also in homogeneous, translation invariant systems ([8, 85, 137], Ref. [118] introduces a location-dependent stimulation that breaks this symmetry), whereas the partial locking found in pulse-coupled systems [154] was induced by inhomogeneities [154].

Since in each biological neural system one particular network is selected that generates a desired dynamics (and function) an open question is whether and in which aspects networks may be optimized, for instance structurally. First examples [103, 104] show that even very sparse heterogeneously coupled networks and very dense, homogeneously coupled networks may be capable of generating the same predefined pattern, cf. Fig. 13.7.

13.6. Conclusions and Open Questions

The currently debated question under which conditions and how patterns of precisely timed spikes and microscopic synchrony may emerge in neural circuits is still far from being answered and also their potential functional role is explored further. On the path towards a final conclusion, experimental and theoretical findings need to be jointly evaluated in a critical way in order to generate and confirm (or reject) key hypotheses. We have here presented two major hypotheses for the mechanism underlying the generation of spike patterns; one where feed-forward anatomy is crucial, and one asserting that spike patterns may emerge collectively in recurrent networks. Up to now, both hypotheses have been neither confirmed nor rejected experimentally and on theoretical grounds, both seem possible.

The hypotheses on synfire and recurrent mechanisms mark only the extreme starting points for such investigations, working in the limits of strong feed-forward chains and of no particular coarse structure, respectively. Intermediate possibilities need to be explored as well. For instance, what is the impact of observed non-random topology, such as motifs present in otherwise apparently randomly connected, spatially extended circuits [103, 107, 123, 141, 142]?

On the one hand, one may be tempted to argue that in model studies on the emergence of spike patterns more and more biological details need to be taken into account. For instance, only few works so far take into account additional dynamical features such as synaptic plasticity, synaptic failure and intrinsic noise or dendritic non-linearities (e.g. [68, 106, 112]), among others. On the other hand, model reduction is essential to isolate potential mechanisms that underlie any hypothesis and thus to restrict a hypothesis as strongly as possible to make it experimentally testable. The theoretical analyses of feed-forward chains of spiking neurons already raise many non-trivial problems, and even the idealized recurrent neural network models considered above typically exhibit highly complex dynamics. Idealized models enable us, nevertheless, to understand mechanisms in feed-forward and even in recurrent networks in a systematic way.

And indeed, recent studies of reduced models for instance revealed that non-additive dendritic integration can support the propagation of synchronous spiking activity in recurrent networks even if they are purely randomly connected and do not contain additional feed-forward connectivity [106]. Conceptually, this constitutes another cornerstone for bridging the gap between feed-forward and recurrent perspectives because propagation of synchronous activity similarly underlies the emergence of spike patterns in both hypotheses. At the same time, propagation of synchronous activity in random recurrent networks might be experimentally distinguished from that along feed-forward chains: In networks containing groups of neurons with specific feed-forward connections between them, synchronous activity propagates along paths that are predefined by that anatomy such that propagation takes place with high probability along the same paths if the experiment is repeated with the same inital group of neurons synchronized. In contrast, in random recurrent networks without specific feed-forward structures but with non-additive dendritic features, synchrony propagates along self-organized paths that may vary among repetitive trials of an experiments. This mechanistic difference implies different types of possible patterns of precisely timed spikes and thus a dynamics that offers an experimental distinction between the two hypotheses.

The balanced activity described in section 13.4.3 constitutes another example where simple models of neural circuits substantially helped to understand the biological network dynamics. The balanced state was first investigated for binary-state neuron models [162] and the mechanism that generates its irregular spiking behavior points to a collective network effect that is due to simultaneously strong and only weakly correlated inhibitory and excitatory inputs. Due to this basic network mechanism, such balanced activity robustly occurs across different systems; it is prevalent also for biophysically more detailed models and provides the first consistent explanation of the irregularity observed in biological neural circuits. Perhaps reduced models will similarly help to better understand the conditions under which spike patterns emerge in feed-forward and in recurrent networks.

When physiological experiments in the near future pin down in which systems patterns of precisely timed spikes definitely carry information that is not contained in the spike rate, the question of their functional role and their actual origin becomes more specific – and even more urgent. In particular if linked with further extensive theoretical studies on all levels, on stochastic and deterministic, abstract and biophysically detailed models, on mechanistic single neuron aspects and on network effects, such experimental investigations are likely to give key answers to the question how the brain computes.

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