S.1 Time-averaged weight change in fully connected assemblies

The correlation function of our Poisson model neurons reads in frequency space

$$\tilde{C}(\omega) = \tilde{C}^0(\omega) + \tilde{C}^1(\omega) = 2\pi\delta(\omega) r r^T + \left(\mathbb{1} - \tilde{a}(\omega)W\right)^{-1} D \left(\mathbb{1} - \tilde{a}(-\omega)W^T\right)^{-1}, \tag{S.1}$$

see main text Eq. 4. For a homogeneously coupled assembly of N neurons with recurrent weights $\hat{w}$ and identical spontaneous rates λ_0, all rates r_i are the same,

$$r_i =: r = \frac{\lambda_0}{1 - (N-1)\hat{w}}, \tag{S.2}$$

see Eq. 13. This fixes the first right hand side term of Eq. S.1 and $D = r\mathbb{1}$. W has zeros on the diagonal and otherwise entries $\hat{w}$. The second right hand side term of Eq. S.1 can thus be written as

$$\begin{aligned} \tilde{C}^1(\omega) &= \left(\mathbb{1} - \tilde{a}(\omega)W\right)^{-1} D \left(\mathbb{1} - \tilde{a}(-\omega)W^T\right)^{-1} \\ &= r\left(\alpha_+\mathbb{1} + \beta_+ J_N\right)^{-1}\left(\alpha_-\mathbb{1} - \beta_+ J_N\right)^{-1}, \end{aligned} \tag{S.3}$$

where J_N is the $N \times N$ matrix of ones, $\alpha_\pm = 1 + \hat{w}\tilde{a}(\pm\omega)$, and $\beta_\pm = -\hat{w}\tilde{a}(\pm\omega)$. We obtain the inverses of the matrices by assuming that they have the same general structure, that is using the ansatz

$$\left(\alpha_\pm\mathbb{1} + \beta_\pm J_N\right)^{-1} = \gamma_\pm\mathbb{1} + \delta_\pm J_N. \tag{S.4}$$

This yields

$$\gamma_\pm = \frac{1}{\alpha_\pm} = \frac{1}{1 + \hat{w}\tilde{a}(\pm\omega)}, \tag{S.5}$$

$$\delta_\pm = \frac{-\beta_\pm}{\alpha_\pm(\alpha_\pm + N\beta_\pm)} = \frac{\hat{w}\tilde{a}(\pm\omega)}{(1 + \hat{w}\tilde{a}(\pm\omega))(1 - \hat{w}\tilde{a}(\pm\omega)(N-1))}. \tag{S.6}$$

In terms of $\gamma_\pm$ and $\delta_\pm$, $C^1(\omega)$ reads

$$\begin{aligned} \tilde{C}^1(\omega) &= r\left(\gamma_+\mathbb{1} + \delta_+ J_N\right)\left(\gamma_-\mathbb{1} + \delta_- J_N\right) \\ &= r\left(\gamma_+\gamma_-\mathbb{1} + \gamma_+\delta_- J_N + \gamma_-\delta_+ J_N + N\delta_+\delta_- J_N\right). \end{aligned} \tag{S.7}$$

Since there are no autapses in the network, we are only interested in the off-diagonal elements,

$$\tilde{C}^1_{i\neq j}(\omega) = r\left(\gamma_+\delta_- + \gamma_-\delta_+ + N\delta_+\delta_-\right). \tag{S.8}$$

The Fourier transform of the synaptic kernel Eq. 2 is

$$\tilde{a}(\omega) = \frac{1}{1 + i\omega\tau_\mathrm{s}}, \tag{S.9}$$

such that

$$\begin{aligned} \tilde{C}^1_{i\neq j}(\omega) = r\Bigg(& \frac{\hat{w}(1 + i\omega\tau_\mathrm{s})(1 - i\omega\tau_\mathrm{s})}{(1 + i\omega\tau_\mathrm{s} + \hat{w})(1 - i\omega\tau_\mathrm{s} + \hat{w})(1 - i\omega\tau_\mathrm{s} - (N-1)\hat{w})} + \\ & + \frac{\hat{w}(1 - i\omega\tau_\mathrm{s})(1 + i\omega\tau_\mathrm{s})}{(1 - i\omega\tau_\mathrm{s} + \hat{w})(1 + i\omega\tau_\mathrm{s} + \hat{w})(1 + i\omega\tau_\mathrm{s} - (N-1)\hat{w})} + \\ & + \frac{N\hat{w}^2(1 + i\omega\tau_\mathrm{s})(1 - i\omega\tau_\mathrm{s})}{(1 + i\omega\tau_\mathrm{s} + \hat{w})(1 + i\omega\tau_\mathrm{s} - (N-1)\hat{w})(1 - i\omega\tau_\mathrm{s} + \hat{w})(1 - i\omega\tau_\mathrm{s} - (N-1)\hat{w})}\Bigg). \end{aligned} \tag{S.10}$$

For the Fourier transform of the plasticity window Eq. 5 we have

$$\tilde{F}(\omega) = \frac{2A_\mathrm{p}\tau_\mathrm{p}}{(1+i\tau_\mathrm{p}\omega)(1-i\tau_\mathrm{p}\omega)} + \frac{2A_\mathrm{d}\tau_\mathrm{d}}{(1+i\tau_\mathrm{d}\omega)(1-i\tau_\mathrm{d}\omega)} = \tilde{F}(-\omega). \tag{S.11}$$

Inserting Eqs. S.1 and S.2 into Eq. 8 gives

$$\begin{aligned}\overline{\Delta W_{ij}}(N) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\ \tilde{C}_{ij}(\omega)\tilde{F}(-\omega)\\ &= \frac{f_0\lambda_0^2}{(1-(N-1)\hat{w})^2} + \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\ \tilde{C}^1_{ij}(\omega)\tilde{F}(-\omega).\end{aligned} \tag{S.12}$$

Inserting Eqs. S.2, S.10 and S.11 into Eq. S.12 results in

$$\begin{aligned}\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\ \tilde{C}^1_{ij}(\omega)\tilde{F}(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\ &\left(2\pi\frac{\hat{w}(1+i\omega\tau_\mathrm{s})(1-i\omega\tau_\mathrm{s})}{(1+i\omega\tau_\mathrm{s}+\hat{w})(1-i\omega\tau_\mathrm{s}+\hat{w})(1-i\omega\tau_\mathrm{s}-(N-1)\hat{w})}+\right.\\ &+\frac{\hat{w}(1-i\omega\tau_\mathrm{s})(1+i\omega\tau_\mathrm{s})}{(1-i\omega\tau_\mathrm{s}+\hat{w})(1+i\omega\tau_\mathrm{s}+\hat{w})(1+i\omega\tau_\mathrm{s}-(N-1)\hat{w})}+\\ &\left.+\frac{N\hat{w}^2(1+i\omega\tau_\mathrm{s})(1-i\omega\tau_\mathrm{s})}{(1+i\omega\tau_\mathrm{s}+\hat{w})(1+i\omega\tau_\mathrm{s}-(N-1)\hat{w})(1-i\omega\tau_\mathrm{s}+\hat{w})(1-i\omega\tau_\mathrm{s}-(N-1)\hat{w})}\right)\times\\ &\times\left(\frac{2A_\mathrm{p}\tau_\mathrm{p}}{(1+i\tau_\mathrm{p}\omega)(1-i\tau_\mathrm{p}\omega)}+\frac{2A_\mathrm{d}\tau_\mathrm{d}}{(1+i\tau_\mathrm{d}\omega)(1-i\tau_\mathrm{d}\omega)}\right)\frac{\lambda_0}{1-(N-1)\hat{w}}.\end{aligned} \tag{S.13}$$

This integral can be straightforwardly computed using the residue theorem. Together with the zeroth-order term given by Eq. S.12 we obtain the following closed-form expression for the time-averaged weight change:

$$\begin{aligned}\overline{\Delta W_{ij}}(N) = &\frac{2\lambda_0^2(A_\mathrm{p}\tau_\mathrm{p}+A_\mathrm{d}\tau_\mathrm{d})}{(1-(N-1)\hat{w})^2}+\\ &+\frac{\lambda_0 A_\mathrm{p}\tau_\mathrm{p}\hat{w}\left[(2+(2-N)\hat{w})\tau_\mathrm{p}+(2-(N-2)\hat{w}-(N-1)\hat{w}^2)\tau_\mathrm{s}\right]}{(1+\hat{w})(1-(N-1)\hat{w})^2(\tau_\mathrm{s}+(1+\hat{w})\tau_\mathrm{p})(\tau_\mathrm{s}+(1-(N-1)\hat{w})\tau_\mathrm{p})}+\\ &+\frac{\lambda_0 A_\mathrm{d}\tau_\mathrm{d}\hat{w}\left[(2+(2-N)\hat{w})\tau_\mathrm{d}+(2-(N-2)\hat{w}-(N-1)\hat{w}^2)\tau_\mathrm{s}\right]}{(1+\hat{w})(1-(N-1)\hat{w})^2(\tau_\mathrm{s}+(1+\hat{w})\tau_\mathrm{d})(\tau_\mathrm{s}+(1-(N-1)\hat{w})\tau_\mathrm{d})}.\end{aligned} \tag{S.14}$$

S.2 Asymptotic behavior of motif contributions to time-averaged plasticity

We first show that the series expansion Eq. 11 of the time-averaged weight change in a homogeneously connected assembly simplifies to

$$\begin{aligned}\overline{\Delta W_{ij}} &= f_0 r_i r_j + \sum_{\alpha,\beta} f_{\alpha\beta} \sum_m r_m (W^\alpha)_{im} (W^\beta)_{jm} \\ &= f_0 r^2 + \frac{r}{N} \sum_{k=1}^{\infty} f_k \left((N-1)^k - (-1)^k \right) \hat{w}^k, \end{aligned} \tag{S.15}$$

where

$$f_k := \sum_{\alpha+\beta=k} f_{\alpha\beta}. \tag{S.16}$$

In homogeneous assemblies we have $r_i = r$ and $W = \hat{w}(J_N - \mathbb{1})$. Eq. 8 thus yields

$$\begin{aligned}\overline{\Delta W_{ij}} &= f_0 r^2 + r \sum_{\alpha,\beta} f_{\alpha\beta} \left(J_N - \mathbb{1} \right)_{ij}^{\alpha+\beta} \hat{w}^{\alpha+\beta} \\ &= f_0 r^2 + r \sum_{k=1}^{\infty} f_k \left(J_N - \mathbb{1} \right)_{ij}^{k} \hat{w}^{k}. \end{aligned} \tag{S.17}$$

We observe that $J_N^m = N^{m-1} J_N$ for $m \geq 1$ while $J_N^0 = \mathbb{1}$. With this the binomial in Eq. S.17 can be expanded to

$$\begin{aligned}\left(J_N - \mathbb{1} \right)^k &= \sum_{l=0}^{k} \binom{k}{l} J_N^l \left(-\mathbb{1} \right)^{k-l} \\ &= \sum_{l=0}^{k} \binom{k}{l} N^{l-1} J_N (-1)^{k-l} - \frac{(-1)^k}{N} J_N + \left(-\mathbb{1} \right)^k . \end{aligned} \tag{S.18}$$

We are again interested only in off-diagonal elements:

$$\begin{aligned}\left(J_N - \mathbb{1} \right)_{i \neq j}^k &= \sum_{l=0}^{k} \binom{k}{l} N^{l-1} (-1)^{k-l} - \frac{(-1)^k}{N} \\ &= \frac{(N-1)^k - (-1)^k}{N}. \end{aligned} \tag{S.19}$$

Inserting this into Eq. S.17 gives Eq. S.15.

We now show that

$$\lim_{k\to\infty}\sum_{\alpha+\beta=k} f_{\alpha\beta} = \frac{f_0}{2\tau_\mathrm{s}}. \tag{S.20}$$

We start by using Eq. 12, to write

$$\sum_{\alpha+\beta=k} f_{\alpha\beta} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega\ \tilde{F}(-\omega) \sum_{\alpha+\beta=k} \tilde{a}(\omega)^{\alpha}\tilde{a}(-\omega)^{\beta}. \tag{S.21}$$

The sum in the integrand can be rewritten using Eq. S.9,

$$\begin{aligned}
\sum_{\alpha+\beta=k} \tilde{a}(\omega)^{\alpha}\tilde{a}(-\omega)^{\beta} &= \sum_{\alpha=0}^{k} \tilde{a}(\omega)^{\alpha}\tilde{a}(-\omega)^{k-\alpha} \\
&= \sum_{\alpha=0}^{k} \left(\frac{1}{1+i\omega\tau_\mathrm{s}}\right)^{\alpha}\left(\frac{1}{1-i\omega\tau_\mathrm{s}}\right)^{k-\alpha} \\
&= \sum_{\alpha=0}^{k} \left(\frac{1-i\omega\tau_\mathrm{s}}{1+i\omega\tau_\mathrm{s}}\right)^{\alpha}\left(\frac{1}{1-i\omega\tau_\mathrm{s}}\right)^{k} \\
&= \frac{1-\left(\frac{1-i\omega\tau_\mathrm{s}}{1+i\omega\tau_\mathrm{s}}\right)^{k+1}}{1-\left(\frac{1-i\omega\tau_\mathrm{s}}{1+i\omega\tau_\mathrm{s}}\right)}\left(\frac{1}{1-i\omega\tau_\mathrm{s}}\right)^{k} \\
&= \frac{\omega\tau_\mathrm{s}-i}{2\omega\tau_\mathrm{s}}\left(\frac{1}{1-i\omega\tau_\mathrm{s}}\right)^{k} + \frac{\omega\tau_\mathrm{s}+i}{2\omega\tau_\mathrm{s}}\left(\frac{1}{1+i\omega\tau_\mathrm{s}}\right)^{k}.
\end{aligned} \tag{S.22}$$

We substitute $\omega' := \omega\tau_\mathrm{s}$ and insert Eq. S.22 into Eq. S.21:

$$\begin{aligned}
\sum_{\alpha+\beta=k} f_{\alpha\beta} &= \frac{1}{2\pi\tau_\mathrm{s}}\int_{-\infty}^{\infty} \mathrm{d}\omega'\ \tilde{F}(-\omega'/\tau_\mathrm{s})\left(\frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k} + \frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k}\right) \\
&= \frac{1}{2\pi\tau_\mathrm{s}}\int_{-\infty}^{\infty} \mathrm{d}\omega' \int_{-\infty}^{\infty} \mathrm{d}t\ \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}} F(t)\left(\frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k} + \frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k}\right) \\
&= \frac{1}{2\pi\tau_\mathrm{s}}\int_{-\infty}^{\infty} \mathrm{d}t\ F(t)\int_{-\infty}^{\infty} \mathrm{d}\omega'\ \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\left(\frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k} + \frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k}\right).
\end{aligned} \tag{S.23}$$

In the second line we employed the definition of the Fourier transform to substitute $\tilde{F}(-\omega/\tau_\mathrm{s})$. (Alternatively, one could use Plancherel's theorem and the inverse Fourier transform of S.22 to obtain the third line.) The integrand of the inner integral of Eq. S.23 has poles at $\omega' = \pm i$ (the singularity at $\omega' = 0$ is removable). For $t > 0$ ($t < 0$) we can compute the integral using a contour over the upper (lower) half complex plane. The residue theorem then yields

$$\begin{aligned}
&\frac{1}{2\pi\tau_\mathrm{s}}\int_{-\infty}^{\infty} \mathrm{d}t\ F(t)\int_{-\infty}^{\infty} \mathrm{d}\omega'\ \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\left(\frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k} + \frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k}\right) \\
&= -\frac{i}{\tau_\mathrm{s}}\int_{-\infty}^{0} \mathrm{d}t\ F(t)\,\mathrm{Res}\left(\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\ \frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k}, -i\right) + \frac{i}{\tau_\mathrm{s}}\int_{0}^{\infty} \mathrm{d}t\ F(t)\,\mathrm{Res}\left(\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\ \frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k}, i\right)
\end{aligned} \tag{S.24}$$

We use that

$$\mathrm{Res}(g,c) = \frac{1}{(k-1)!}\lim_{z\to c}\frac{\mathrm{d}^{k-1}}{\mathrm{d}z^{k-1}}\left((z-c)^k g(z)\right) \tag{S.25}$$

if $g(z)$ has a kth order pole at $z=c$. For $\alpha+\beta=k+1$ the residue at $\omega'=+i$ in Eq. S.24 thus becomes

$$\begin{aligned}
\mathrm{Res}\left(\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\frac{\omega'+i}{2\omega'}\left(\frac{1}{1+i\omega'}\right)^{k+1}, i\right) &= \frac{1}{k!}\lim_{\omega'\to i}\frac{\mathrm{d}^k}{\mathrm{d}\omega'^k}\left((\omega'-i)^{k+1}\left(\frac{1}{1+i\omega'}\right)^{k+1}\left(\frac{1}{2}+\frac{i}{2\omega'}\right)\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\right)\\
&= \frac{1}{i^{k+1}k!}\lim_{\omega'\to i}\frac{\mathrm{d}^k}{\mathrm{d}\omega'^k}\left(\left(\frac{1}{2}+\frac{i}{2\omega'}\right)\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\right)\\
&= \frac{1}{i^{k+1}k!}\lim_{\omega'\to i}\left(\frac{1}{2}\left(\frac{it}{\tau_\mathrm{s}}\right)^k \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}} + \sum_{j=0}^{k}\binom{k}{j}\left(\frac{it}{\tau_\mathrm{s}}\right)^j \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\frac{(-1)^{k-j}(k-j)!\,i}{2\omega'^{k-j+1}}\right)\\
&= \frac{1}{i^{k+1}}\lim_{\omega'\to i}\left(\frac{1}{2k!}\left(\frac{it}{\tau_\mathrm{s}}\right)^k \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}} + \sum_{j=0}^{k}\left(\frac{it}{\tau_\mathrm{s}}\right)^j \mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\frac{(-1)^{k-j}\,i}{2\,j!\,\omega'^{k-j+1}}\right)\\
&= \frac{\mathrm{e}^{-\frac{t}{\tau_\mathrm{s}}}}{2ik!}\left(\frac{t}{\tau_\mathrm{s}}\right)^k + \frac{1}{2i}\mathrm{e}^{-\frac{t}{\tau_\mathrm{s}}}\sum_{j=0}^{k}\frac{1}{j!}\left(\frac{t}{\tau_\mathrm{s}}\right)^j .
\end{aligned} \tag{S.26}$$

Similarly for the residue at $\omega'=-i$ we obtain

$$\mathrm{Res}\left(\mathrm{e}^{\frac{i\omega' t}{\tau_\mathrm{s}}}\frac{\omega'-i}{2\omega'}\left(\frac{1}{1-i\omega'}\right)^{k+1}, -i\right) = \frac{i\,\mathrm{e}^{\frac{t}{\tau_\mathrm{s}}}}{2k!}\left(\frac{-t}{\tau_\mathrm{s}}\right)^k + \frac{i}{2}\mathrm{e}^{\frac{t}{\tau_\mathrm{s}}}\sum_{j=0}^{k}\frac{1}{j!}\left(\frac{-t}{\tau_\mathrm{s}}\right)^j . \tag{S.27}$$

Inserting Eq. S.26 and Eq. S.27 into Eq. S.24 gives

$$\sum_{\alpha+\beta=k} f_{\alpha\beta} = \frac{1}{\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\ F(t)\left(\frac{\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}}{2(k-1)!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^{k-1} + \frac{1}{2}\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}\sum_{j=0}^{k-1}\frac{1}{j!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^j\right) \tag{S.28}$$

$$=: \frac{1}{\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\ F(t)A_k(t). \tag{S.29}$$

We then take the limit $k\to\infty$:

$$\begin{aligned}
\lim_{k\to\infty}\sum_{\alpha+\beta=k} f_{\alpha\beta} &= \lim_{k\to\infty}\frac{1}{\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\ F(t)\left(\frac{\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}}{2(k-1)!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^{k-1} + \frac{1}{2}\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}\sum_{j=0}^{k-1}\frac{1}{j!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^j\right)\\
&= \lim_{k\to\infty}\frac{1}{\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\ F(t)\frac{\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}}{2(k-1)!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^{k-1} + \frac{1}{2\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\ F(t)\ \mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}\lim_{k\to\infty}\sum_{j=0}^{k-1}\frac{1}{j!}\left(\frac{|t|}{\tau_\mathrm{s}}\right)^j .
\end{aligned} \tag{S.30}$$

The first limit vanishes as long as $F(t)$ goes to zero polynomially or faster for large $|t|$. This is guaranteed if $F(t)$ is integrable over $(-\infty,\infty)$, which we already implicitly assume for example to do the Fourier transform. The limit of the power series in the second term is the exponential function. (Limit and integral are there interchangeable due to the dominated convergence theorem.) From Eq. S.30 we thus obtain the final result:

$$\begin{aligned}\lim_{k\to\infty}\sum_{\alpha+\beta=k} f_{\alpha\beta} &= \frac{1}{2\tau_\mathrm{s}}\int_{-\infty}^{\infty}\mathrm{d}t\; F(t)\;\mathrm{e}^{-\frac{|t|}{\tau_\mathrm{s}}}\,\mathrm{e}^{\frac{|t|}{\tau_\mathrm{s}}}\\ &= \frac{f_0}{2\tau_\mathrm{s}}.\end{aligned} \tag{S.31}$$

To develop an intuition about this result, we observe that the inner integral of Eq. S.23 is up to a constant factor the inverse Fourier transformation of $\sum_{\alpha+\beta=k}\tilde{a}(\omega)^\alpha\tilde{a}(-\omega)^\beta$. Using its linearity we may apply the inverse Fourier transform to each summand individually. This yields $a(t)$ convolved α times with itself and with $a(-t)$ convolved β times with itself. Due to Eq. 2 this equals the probability distribution of a sum of α exponentially distributed independent stochastic variables with mean τ_s minus the sum of β stochastic variables of the same type. In other words, we have the probability distribution of a spike time occurring at the end of a cascade of α spikes with exponentially distributed inter-spike intervals minus the time of a spike occurring at the end of a similar cascade of β spikes. This reflects the fact that the term $f_{\alpha\beta}$ covers the impact of the motif where a neuron evokes a spike in the pre- and postsynaptic neurons after spike cascades of length β and α. The probability distribution has mean $(\alpha-\beta)\tau_s$ and standard deviation $\sqrt{k}\tau_s$. The standard deviations of all distributions are thus identical and neighboring distributions have distance $2\tau_s$. According to the central limit theorem, for large k the distributions approximate normal distributions. For α and β adding to the same k, these are equidistantly shifted but otherwise identical. With increasing k they broaden, such that their superposition forms a plateau. The increase in overlap thereby compensates the decrease in the distribution heights. The number of distributions increases with increasing k. The added distributions, however, do not lead to a non-compensatory increase of the superposition in the relevant center where it overlaps with F. This is because the added distributions initially do not reach the center, as their mean scales with k while their width scales only with $\sqrt{k}$.

S.3 Parameters

	N	$\hat{w}$	τ_{s}	A_{p}	A_{d}	τ_{p}	τ_{d}	μ	λ_0
Fig. 2	120	0.04	0.01 s	0.08	-0.053	0.025 s	0.05 s	0.07	0.15 Hz
Fig. 3(a)			0.01 s	0.08	-0.053	0.025 s	0.05 s		0.15 Hz
Fig. 4	80	0.026	0.01 s	0.08	-0.066	0.035 s	0.05 s	0.07	0.15 Hz
Fig. 5	120	0.017	0.01 s	0.08	-0.044	0.022 s	0.055 s	0.02	0.15 Hz
Fig. 6(a)	60	0.0158	0.01 s	0.08	-0.044	0.022 s	0.055 s	0.021	0.15 Hz
Fig. 6(b)	60	0.024	0.01 s	0.08	-0.053	0.025 s	0.05 s	0.045	0.15 Hz
Fig. 7	190	0.018	0.01 s	0.08	- 0.042	0.026 s	0.065 s	0.017	0.15 Hz
Fig. 8(a)	72	0.056	0.01 s	0.08	-0.053	0.025 s	0.05 s		0.2 Hz
Fig. 8(b)	120	0.024	0.01 s	0.08	-0.053	0.025 s	0.05 s	0.05	0.15 Hz
Fig. 9(a)			0.01 s	0.08	-0.053	0.025 s	0.05 s		0.15 Hz
Fig. 9(b)	120	0.058	0.01 s	0.08	-0.053	0.025 s	0.05 s	0.07	0.15 Hz
Fig. 9(c)	150		0.01 s	0.08	-0.053	0.025 s	0.05 s		0.2 Hz
Fig. S1	80	0.024	0.01 s	0.08	-0.053	0.025 s	0.05 s	0.055	0.15 Hz
Fig. S2				0.08	-0.053	0.025 s	0.05 s		
Fig. S3	72	0.056	0.01 s	0.08	-0.053	0.025 s	0.05 s		0.2 Hz

Fig. 2: The networks starts with an initially random connectivity where each weight is independently and uniformly drawn from $[0, 0.25\hat{w}]$.
Fig. 4: The initial weight matrix has an assembly of neurons 1-20 interconnected with $W_{ij} = \hat{w}$ and background connectivity as in Fig. 2. During the stimulation period neurons 21-40 are given input from a single source of Poisson spiking with $\lambda = 4.18\,\mathrm{Hz}$ and input weights $w_{\mathrm{in}} = 10\hat{w}$.
Fig. 6a: The neuron with index 10 has spontaneous rate $\lambda_0 = 0.08\,\mathrm{Hz}$.
Fig. 6b: The neuron with index 10 has spontaneous rate $\lambda_0 = 0.01\,\mathrm{Hz}$.
Fig. 8a: The initial weight matrix is as in Fig. 2 but with weights drawn from $[0, 0.15\hat{w}]$.
Fig. 8b: Every 3×10^5 s a neuron changes its spontaneous rates with probability $p = 0.03$ to $\lambda_0' = 0.03\,\mathrm{Hz}$ for 3×10^5 s. For the last 3×10^5 s of the simulation all spontaneous rates are kept at $\lambda_0 = 0.15\,\mathrm{Hz}$.

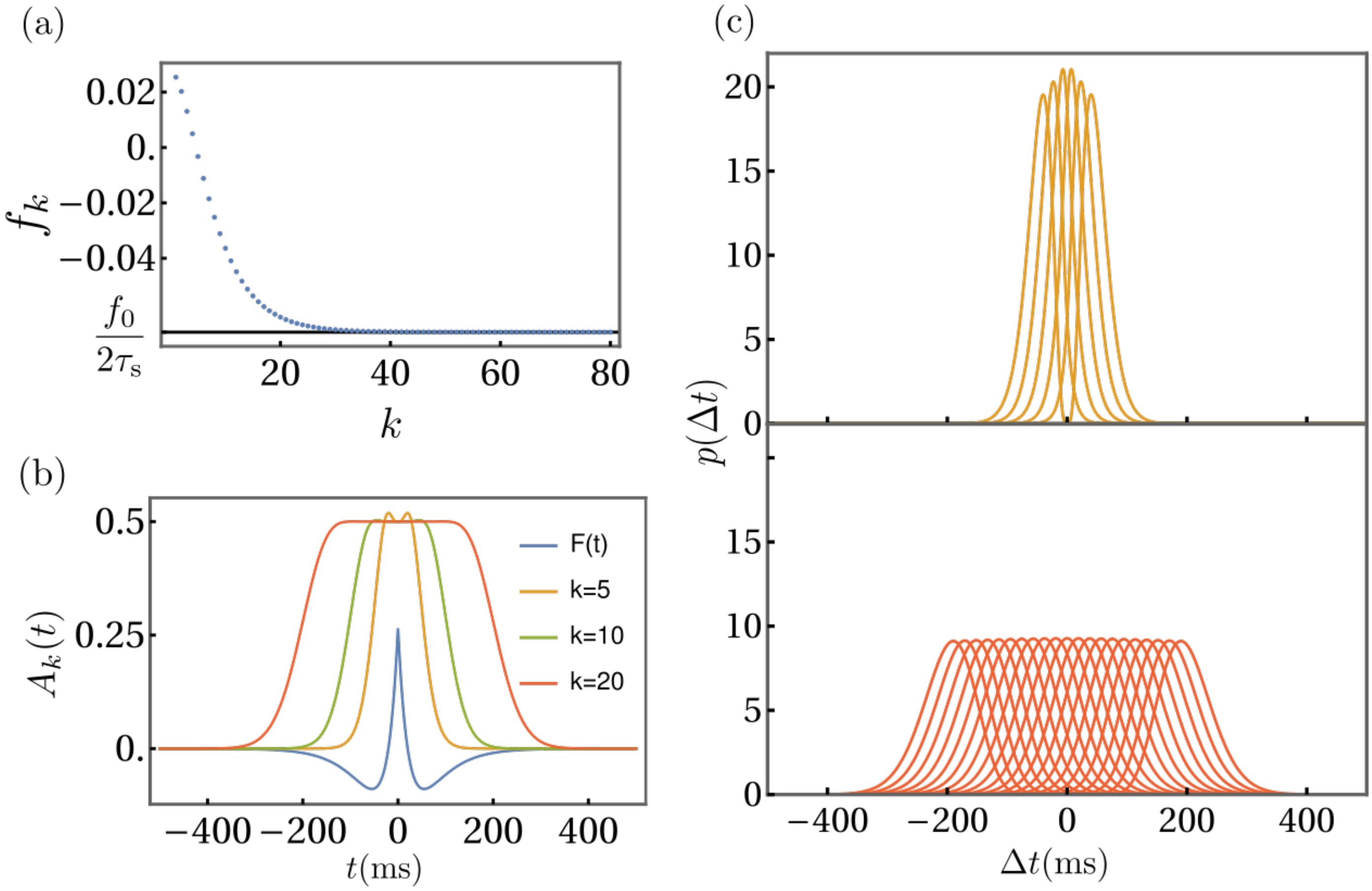

Figure S1: Asymptotic behavior of sums f_k of motif contributions. (a) Convergence of f_k to $\frac{f_0}{2\tau_s}$ for high orders of k. (b) Sums of convolutions of the synaptic current functions $A_k(t)$ (see Eq. S.29), for different values of k. As discussed in section S.2, these curves can be interpreted as resulting from the superposition of distributions of time-lags for spike cascades of total length k affecting pre- and postsynaptic neurons (see (c)). For high orders, $A_k(t)$ becomes a plateau of height $\frac{1}{2}$. The learning window (blue, a.u.), is shown to illustrate the time-scales. (c) Probability distributions of the time lags Δt between the last spikes of two spike-cascades of different lengths β and α. The total cascade length $k = \alpha + \beta$ equals 5 spikes (upper subpanel) or 20 spikes (lower subpanel). For larger k the distributions have larger variance and are more spread out. Their superpositions therefore assume the widening plateau shapes displayed in (b).

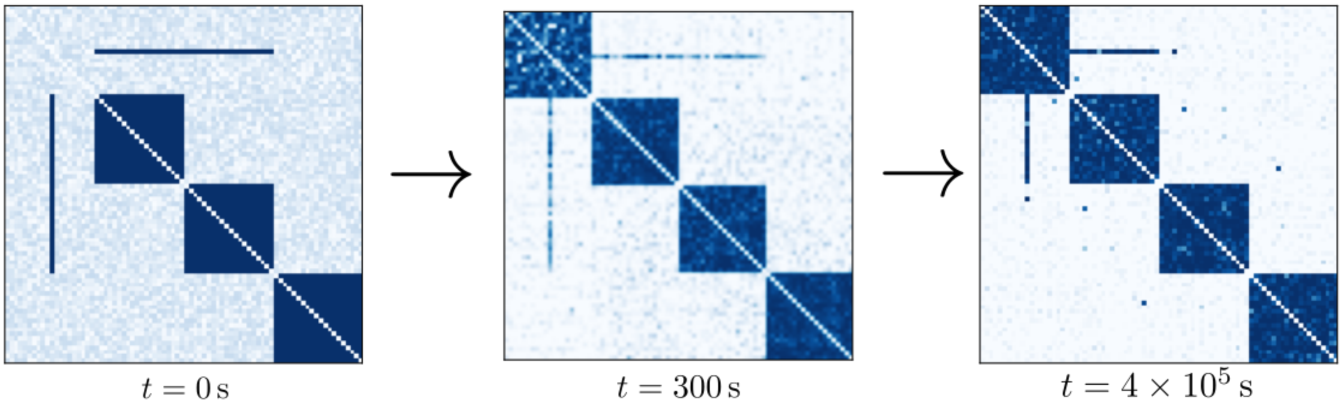

Figure S2: A network with preexisting assemblies, background neurons and a neuron that is part of two assemblies, learns a new assembly. The figure displays the weight matrices of the initial network configuration (left hand side), after stimulation (middle) and after a longer time (right hand side). The stimulation of neurons 1-20 lasts for 300 s with the same stimulation protocol as in Fig. 4. The initially strong connections of the "overlap neuron" (neuron 10) with two other assemblies experience depression after the neuron is recruited to the new assembly (middle). Subsequently it loses its connections to one of the assemblies it previously belonged to (right hand side).

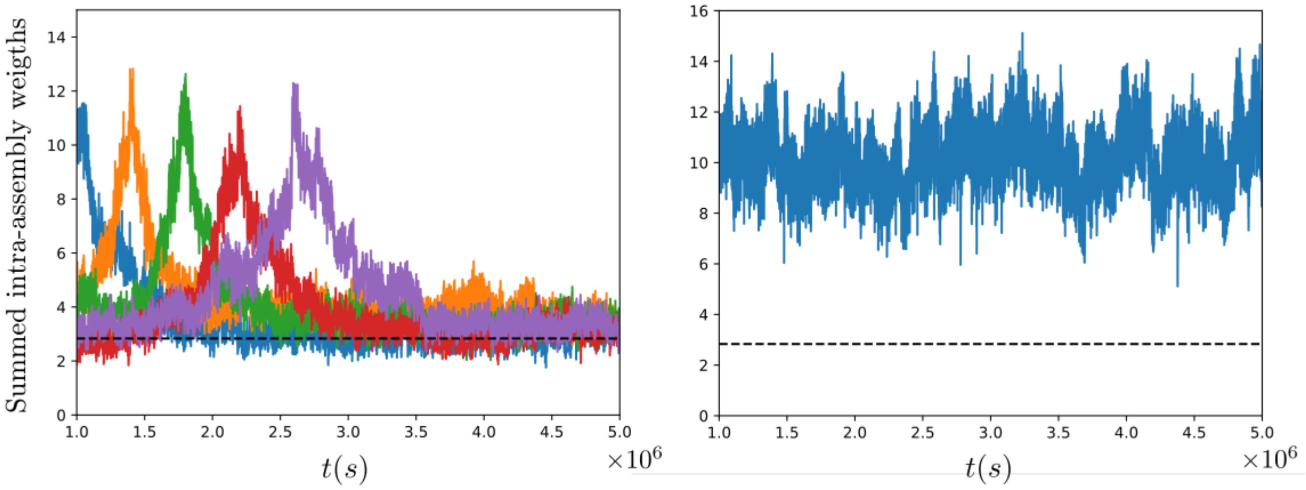

Figure S3: Drifting assembly dynamics. (a) The network with drifting assemblies of Fig. 8a is simulated over a longer time. We choose at five time points ($t = 1.0 \times 10^6\,\mathrm{s}, 1.4 \times 10^6\,\mathrm{s}, 1.8 \times 10^6\,\mathrm{s}, 2.2 \times 10^6\,\mathrm{s}, 2.6 \times 10^6\,\mathrm{s}$) an assembly as a reference and note the neurons forming it. Thereafter, we track the sum of weights between these neurons in the present, future and past weight matrices. For sufficiently long temporal distances this sum approximately reaches chance level (dashed black line, for an average size assembly), which we define as the average of the sum of interconnections in groups of randomly chosen neurons. We can therefore conclude that assemblies indeed completely drift, that is, there is no constant "stable core" set of neurons in an assembly. (b) Tracking of a single drifting assembly over time. At $t_0 = 1 \times 10^6\,\mathrm{s}$ we choose assembly 1 (of the four assemblies) and set $t = t_0$. We then compute the sum of the weights at time $t + \Delta t$ ($\Delta t = 2\,\mathrm{s}$) between the neurons that formed assembly 1 at time t, to see if they are still strongly connected. We repeat the procedure using the actual assembly 1 at the new time $t = t_0 + \Delta t$ (which may have exchanged individual neurons compared to the one at t_0). In particular, we again compute the weights between its neurons at $t + \Delta t$ and so on. We find that the sum of weights stays at a consistently high level. In other words, neurons that form assembly 1 at a time t are still strongly connected at $t + \Delta t$. The change of the assembly is therefore gradual; most neurons that are part of the assembly at t are still part of it at $t + \Delta t$. This implies that the assembly can be tracked over time, despite the complete drift.