

SUPPLEMENTAL MATERIAL

Brain-wide representational drift: memory consolidation and entropic force

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Supplementary figures

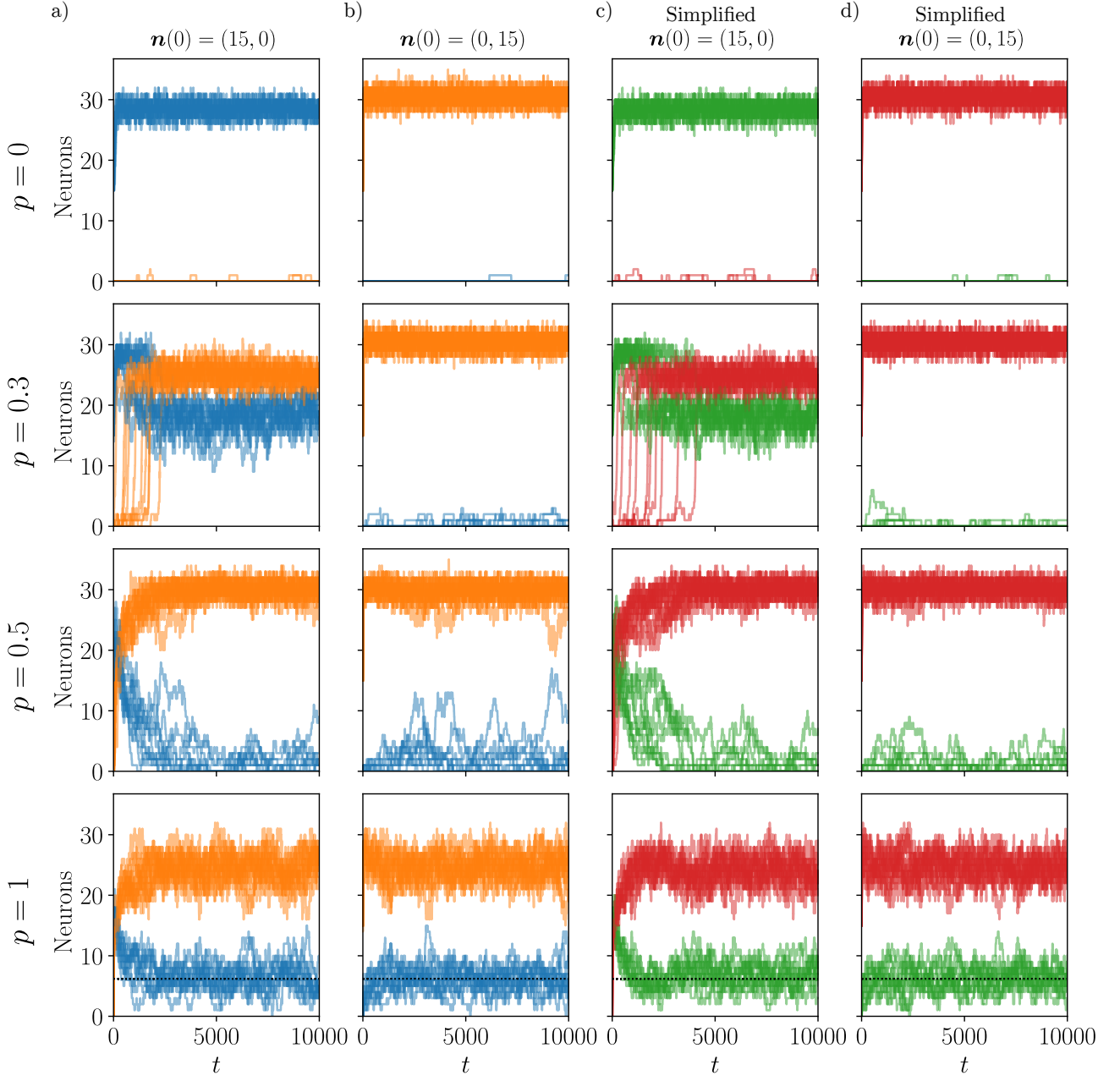


Figure S1: Engram macrostate dynamics in the random-and-deterministic drift model with full and simplified energy function (parameters as in main text Fig. 4). Left hand side columns a,b: dynamics in the model with engram energy given by main text Eq. 2. Right hand side columns c,d: dynamics in the simplified model with engram energy given by main text Eq. 3. Columns a,c: engram initially in Region 1; columns b,d: engram initially in Region 2. Rows: various inter-region connectivity probabilities. The number of neurons in Region 1 is displayed in blue (full model) or green (simplified model), the number of neurons in Region 2 in orange (full model) or red (simplified model). Ten realizations are shown for each initial condition and probability. The dynamics in both models are qualitatively similar.

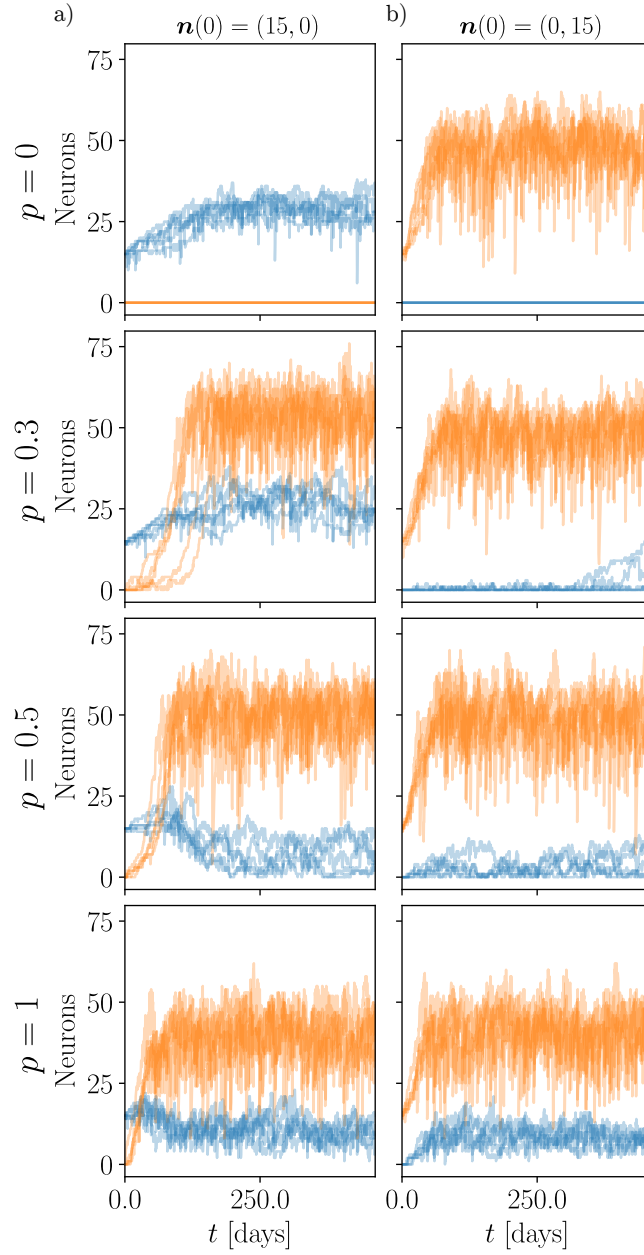


Figure S2: Engram macrostate dynamics in the biologically detailed model. Column a: engram initially in Region 1; columns b,d: engram initially in Region 2. Rows: various inter-region connectivity probabilities. The number of neurons in Region 1 is displayed in blue, the number of neurons in Region 2 in orange. We observe qualitatively similar forms of dynamics as in the random-and-deterministic drift model (cf. Fig. S1). There are also obvious differences: in particular the noise level in the biologically more detailed model is generally higher.

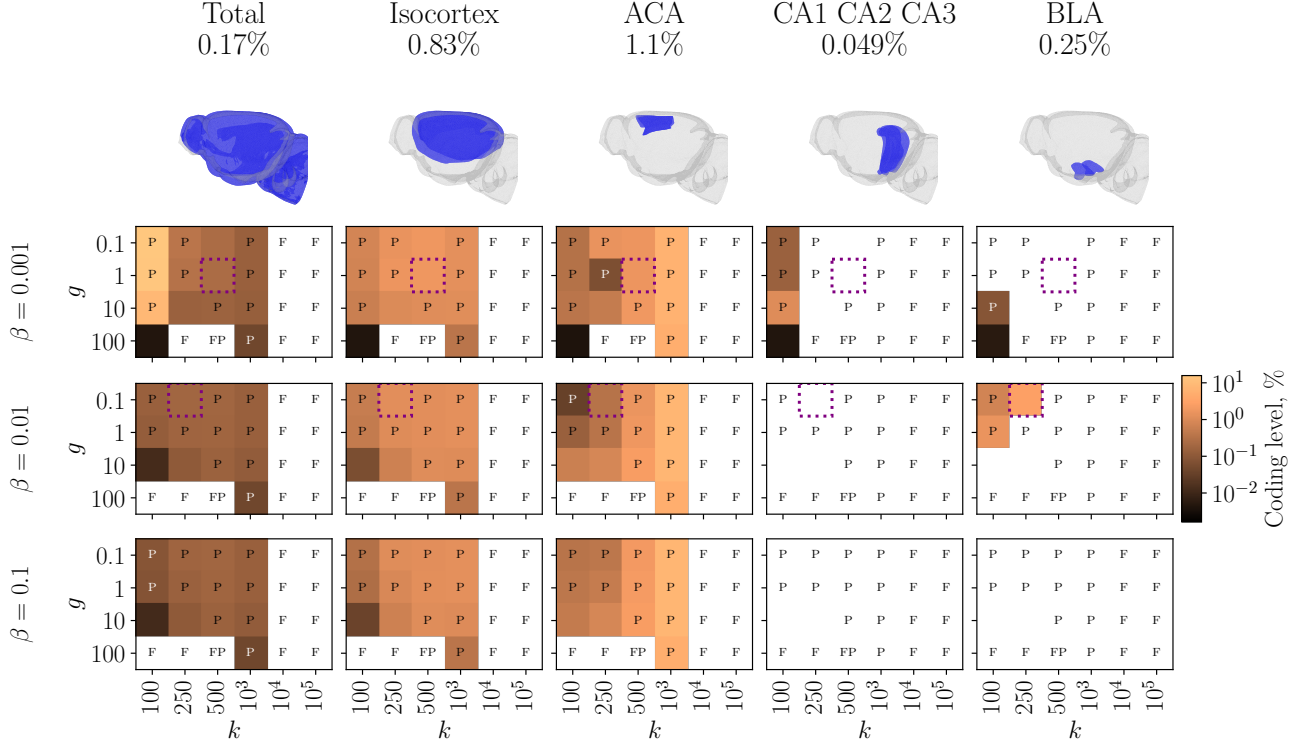


Figure S3: Quasi-equilibria of the fear memory engram in the mouse brain for different parameters of the energy. Columns: selected regions. Top row: schematic display of the region along with its initial coding level. Subsequent rows: Various parameters β . Matrices: Various parameters g and k . Matrix entries: quasi-equilibrium coding levels, averaged over realizations. Zero coding level is shown in white. Purple-dashed squares indicate parameters shown in the main text Fig. 6. F - forgetting; P - pathological evolution; BLA - basolateral amigdala; ACA - anterior cingulate area; CA1, CA2, CA3 - hippocampal fields.

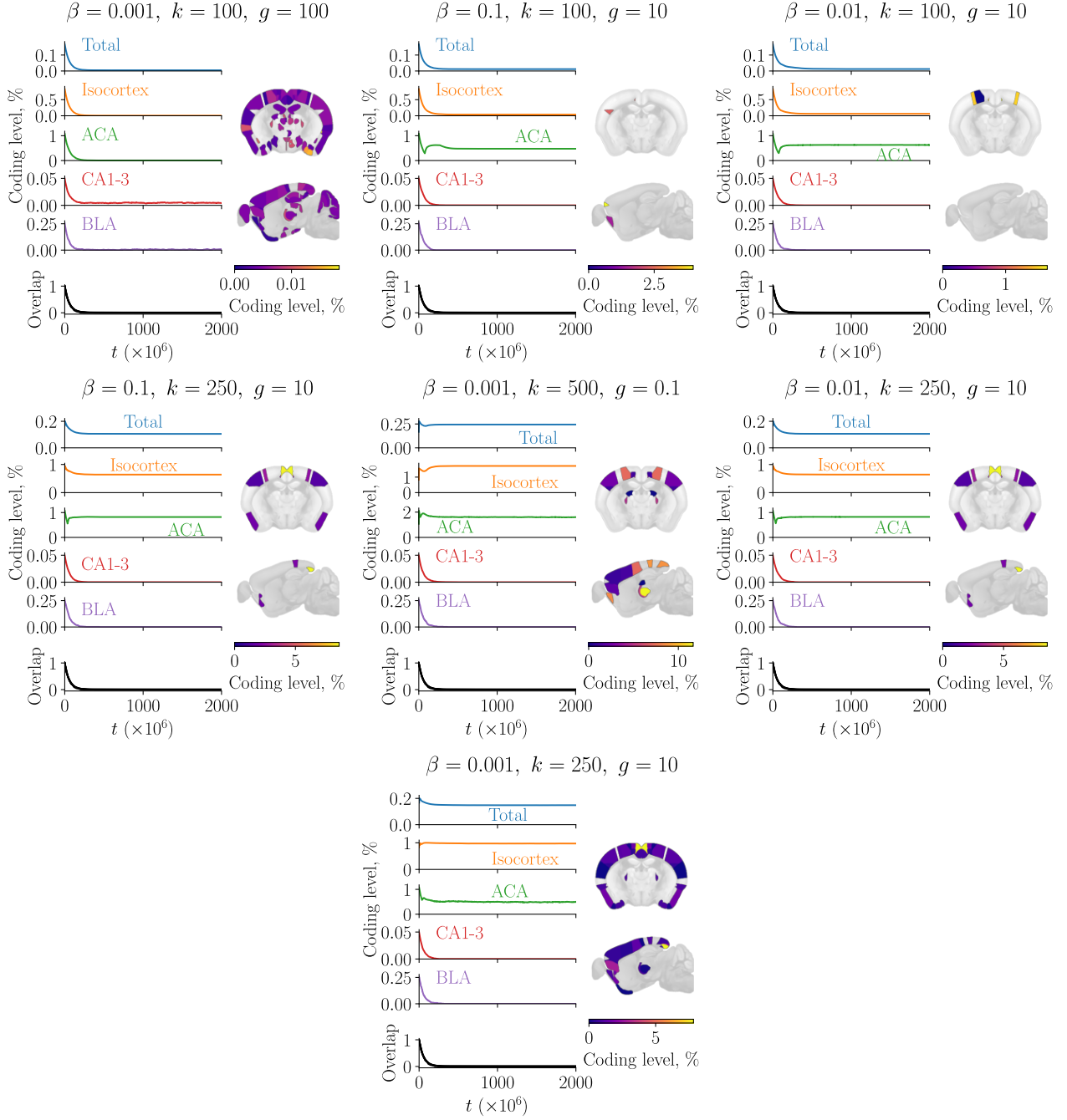


Figure S4: Fear memory engram dynamics and quasi-equilibria in the mouse brain as in main text Fig. 6, for the remaining valid parameter sets.

1 Derivation of the average trajectory

Main text Eq. 9 may be derived as follows:

$$\begin{aligned}
\langle n_1(u)|n_1(t) \rangle &= \sum_{n_1(u)=0}^n n_1(u) P(n_1(u)|n_1(t)) \\
&= \sum_{n_1(u)=0}^n n_1(u) \sum_{n_1(u-1)=0}^n P(n_1(u)|n_1(u-1)) P(n_1(u-1)|n_1(t)) \\
&= \sum_{n_1(u-1)=0}^n \langle n_1(u)|n_1(u-1) \rangle P(n_1(u-1)|n_1(t)) \\
&= \sum_{n_1(u-1)=0}^n ((1-B)n_1(u-1) + A) P(n_1(u-1)|n_1(t)) \\
&= (1-B) \langle n_1(u-1)|n_1(t) \rangle + A,
\end{aligned} \tag{1}$$

2 Derivation of the average energy

In the following we average main text Eq. 10 term by term. Since $A_{ij} \in \{0, 1\}$ are independent Bernoulli random variables, we have $\overline{A_{ij}} = \overline{A_{ij}^2} = p_{ij}$. Furthermore, $\overline{A_{ij}A_{gh}} = p_{ij}p_{gh}$ if either $i \neq g$ or $j \neq h$ or both. We assume that all neurons in region s have the same probability of being structurally connected to neurons in region r , that is $p_{ij} = p_{sr}$ for neuron i in region s and neuron j in region r . We average the first term as follows

$$\begin{aligned}
\sum_s \sum_{i \in s} \left(\overline{\sum_r \sum_{j \in r} A_{ij} m_j - k} \right)^2 m_i &= \sum_{s,r,q} \sum_{\substack{i \in s \\ j \in r \\ h \in q}} \overline{A_{ij} A_{ih}} m_i m_j m_h - 2k \sum_{s,r} \sum_{\substack{i \in s \\ j \in r}} \overline{A_{ij}} m_i m_j \\
&\quad + k^2 \sum_s \sum_{i \in s} m_i \\
&= \sum_{\substack{s,r,q \\ r \neq q}} \sum_{\substack{i \in s \\ j \in r \\ h \in q}} p_{ij} p_{ih} m_i m_j m_h + \sum_{s,r} \sum_{\substack{i \in s \\ j, h \in r \\ j \neq h}} p_{ij} p_{ih} m_i m_j m_h \\
&\quad + \sum_{s,r} \sum_{\substack{i \in s \\ j \in r}} p_{ij} m_i m_j^2 - 2k \sum_{s,r} \sum_{\substack{i \in s \\ j \in r}} p_{ij} m_i m_j \\
&\quad + k^2 \sum_s \sum_{i \in s} m_i \\
&= \sum_{\substack{s,r,q \\ r \neq q}} p_{sr} p_{sq} n_s n_r n_q + \sum_{s,r} p_{sr}^2 n_s n_r (n_r - 1) \\
&\quad + \sum_{s,r} p_{sr} n_s n_r - 2k \sum_{s,r} p_{sr} n_s n_r + k^2 \sum_s n_s \\
&= \sum_s \left(\sum_r p_{sr} n_r - k \right)^2 n_s + \sum_{s,r} p_{sr} (1 - p_{sr}) n_s n_r.
\end{aligned} \tag{2}$$

For averaging the second term note that it is zero if $i = j$, so we need to consider only the case $i \neq j$

$$\begin{aligned}
g \sum_{s,r} \sum_{\substack{i \in s \\ j \in r}} \overline{(A_{ij} - A_{ji})^2} m_i m_j &= g \sum_{s,r} \sum_{\substack{i \in s \\ j \in r}} \left(\overline{A_{ij}^2} - 2 \overline{A_{ij} A_{ji}} + \overline{A_{ji}^2} \right) m_i m_j \\
&= g \sum_{s,r} \sum_{\substack{i \in s \\ j \in r \\ i \neq j}} (p_{ij} - 2p_{ij}p_{ji} + p_{ji}) m_i m_j \\
&= g \sum_{s,r} (p_{sr} - 2p_{sr}p_{rs} + p_{rs}) \sum_{\substack{i \in s \\ j \in r \\ i \neq j}} m_i m_j \\
&= g \sum_{\substack{s,r \\ s \neq r}} (p_{sr} - 2p_{sr}p_{rs} + p_{rs}) \sum_{\substack{i \in s \\ j \in r}} m_i m_j \\
&\quad + g \sum_s (2p_{ss} - 2p_{ss}^2) \sum_{\substack{i,j \in s \\ i \neq j}} m_i m_j \\
&= g \sum_{\substack{s,r \\ s \neq r}} (p_{sr} - 2p_{sr}p_{rs} + p_{rs}) n_s n_r \\
&\quad + g \sum_s (2p_{ss} - 2p_{ss}^2) n_s (n_s - 1) \\
&= g \sum_{s,r} (p_{sr} - 2p_{sr}p_{rs} + p_{rs}) n_s n_r - 2g \sum_s p_{ss} (1 - p_{ss}) n_s.
\end{aligned} \tag{3}$$

Combining the two terms yields main text Eq. 3.